FORECASTING LANDSLIDE USING ELLIPTIC UMBILIC CATASTROPHE MODEL

PEI QIANG[i) and JINMING XU[ii)

ABSTRACT

Using the available monitoring data, an expression of vertical deformations with associated coordinates in rock slope was established, in which coefficients were obtained by the least square method. After transforming the expression into the standard form of elliptic umbilic catastrophe model, the stability criterion and initial unstable points for a rock slope were determined with the statistically analysis of this model. Calculations in the proposed elliptic umbilic catastrophe model may present more detailed processes and therefore provide more explaining results than those from the commonly-used cusp catastrophe model. Two case studies, respectively referred to the Ca'di Malta landslide in Italy and the Baiheling landslide in China, show the reliability of the method presented in this paper.

Key words: Rock Slope, Elliptic Umbilic Catastrophe Model, Least Square Method, Initial Unstable Point

1 INTRODUCTION

The catastrophe theory [1-2] was established in 1972 by Thom, a France scientist. This theory can be used in studying the discontinuous phenomena, especially those representing the process of the gradual change to the sudden change. Many considerations were paid in literature to the system with little knowledge of internal interactions. Henley [3] and John et al [4] applied the catastrophe theory in studying commonly-exist geological phenomena. Qin et al [5] investigated the mechanisms associated with the instabilities of the system with hard top-slate and coal column using the cusp catastrophe theory. In the study of landslide, Sun et al [6] analyzed the plane-sliding landslide using the swallow-tail catastrophe theory and obtained some available interactions between the internal factors influencing the stability of slopes; Xu et al [7] setup a cusp catastrophe model and presented the failure mechanism for a landslide with shallow layers during strongly-rainfall; Long et al [8] revealed the instability mechanism of the landslide triggered by rainfall using cusp catastrophe model. Liu et al [9] used a elliptic umbilic catastrophe model to predict the position and magnitude of heavy earthquakes.

Due to the complexion of the slope system and the interactions between the influencing factors, it is very difficult to conclude all of the factors in a simple model in analyzing landslide. After considering the external factors (focusing on the displacements of a slope), the expression of the elevation difference with the planar coordinates will be established in this paper, in which the related coefficients are computed using the monitoring data by the least square method. This expression was then transformed into the standard formula of the elliptic umbilic catastrophe model. The stability of the slope was further determined by the associated criterion for the rock slope.

2 METHODOLOGY

2.1 The establishment of the elliptic umbilic catastrophe model

Before sliding, the slight displacement may exist in a rock slope. The landslide is the process first from a stable state to a sudden unstable state and then to an intensely- sliding state. Therefore, the relation of the vertical displacement Δz with the planar coordinates (x,y) in each point of the slope may be expressed as

$$\Delta z = f(x, y)$$

here $f(x,y)$ can be curve-fitted using a three-order polynomial, or

$$\begin{aligned}\Delta z = {} & a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 + \\ & a_{20}x^2 + a_{11}xy + a_{02}y^2 + \\ & a_{10}x + a_{01}y + a_{00}\end{aligned} \tag{1}$$

where a_{30}, a_{21}, a_{12}, a_{03}, a_{20}, a_{11}, a_{02}, a_{10}, a_{01}, and a_{00} are the curve-fit coefficients and may be obtained by the least square method.

After the process presented by Liu et al [9], Eq.(1) may be transformed into the standard form of the elliptic umbilic catastrophe model, or

i) Graduate Student, Department of Civil Engineering, Shanghai University, Shanghai, 200072, China.
ii) Corresponding Author, Associate Professor, Department of Civil Engineering, Shanghai University, Shanghai, 200072, China.

$$\Delta\bar{z} = \bar{x}^3 - \bar{x}\bar{y}^2 + w(\bar{x}^2 + \bar{y}^2) - u\bar{x} + v\bar{y} \tag{2}$$

where

$\Delta\bar{z} = \Delta z + R_{00}$, $\bar{x} = x - c$,

$\bar{y} = y - d$, $u = R_{10}$, $v = R_{01}$,

$$w = R_{20} \tag{3a}$$

$$R_{10} = E_{10} + 2E_{20}c + E_{11}d + 3E_{30}c^2 + 2E_{21}E_{20}cd + E_{12} \tag{3b}$$

$$R_{01} = E_{01} + E_{11}c + 2E_{02}d + E_{20}c^2 + 2E_{12}E_{20}cd + 3E_{03}d^2 \tag{3c}$$

$$R_{20} = E_{20} + 3E_{30}c + E_{21}E_{20}d \tag{3d}$$

$$R_{00} = a_{00} + E_{10}c + E_{01}d + E_{20}c^2 + E_{11}cd + E_{02}d^2 + E_{30}c^3 + E_{21}E_{20}c^2d + E_{12}E_{20}cd^2 + E_{03}d^3 \tag{3e}$$

$E_{01} = a_{10}B_1 + a_{01}B_2$, $E_{10} = a_{10}A_1 + a_{01}A_2$

$$E_{20} = a_{20}A_1^2 + a_{11}A_1A_2 + a_{02}A_2^2 \tag{3f}$$

$$E_{11} = 2a_{20}A_1B_1 + a_{11}A_1B_2 + a_{11}A_2B_1 + 2a_{02}A_2B_2 \tag{3g}$$

$E_{02} = a_{20}B_1^2 + a_{11}B_1B_2 + a_{02}B_2^2$

$$E_{30} = a_{30}A_1^3 + a_{21}A_1^2A_2 + a_{12}A_1A_2^2 + a_{03}A_2^3 \tag{3h}$$

$$E_{21} = 3a_{30}A_1^2B_1 + 2a_{21}A_1B_1A_2 + a_{12}A_2^2B_1 + 2a_{12}A_1A_2B_2 + a_{21}A_1^2B_2 + 3a_{03}A_2^2B_2 \tag{3i}$$

$$E_{12} = 3a_{30}A_1B_1^2 + a_{21}A_2B_1^2 + 2a_{21}A_1B_1B_2 + a_{12}A_1B_2^2 + 2a_{12}A_2B_1B_2 + 3a_{03}A_2B_2^2 \tag{3j}$$

$$E_{03} = a_{30}B_1^3 + a_{21}B_1^2B_2 + a_{12}B_1B_2^2 + a_{03}B_2^3 \tag{3k}$$

$A_1 = e_1A_2$, $B_1 = e_2B_2$, $A_2 = (S/3)^{1/3}$

$$B_2 = (-3S/RT)^{1/2} \tag{3l}$$

$S = a_{30}e_2^3 + a_{21}e_2^2 + a_{12}e_2 + a_{03}$,

$R = a_{20}e_1^2 + a_{11}e_1 + a_{02}$,

$T = 3a_{30}e_1e_2^2 + a_{21}e_2 + a_{12}e_1 + 2a_{21}e_1e_2 + 2a_{12}e_2 + 3a_{03}$

$$c = (A_{10}A_{22} - A_{12}A_{20})/(A_{11}A_{22} - A_{12}A_{21}) \tag{3m}$$

$$d = (A_{11}A_{20} - A_{10}A_{21})/(A_{11}A_{22} - A_{12}A_{21}) \tag{3n}$$

$A_{11} = 3E_{30} - E_{12}E_{20}$, $A_{12} = E_{21}E_{20} - 3E_{03}$,

$A_{10} = E_{02} - E_{20}$, $A_{20} = -E_{11}$, $A_{21} = 2E_{21}E_{20}$,

$A_{22} = 2E_{12}E_{20}$,

Here e_1 and e_2 are the real root of the equation respectively in Eq.(4):

$$\begin{aligned} b_2e_1^2 + b_1e_1 + b_0 &= 0 \\ a_{30}e_2^3 + a_{21}e_2^2 + a_{12}e_2 + a_{03} &= 0 \end{aligned} \tag{4}$$

where

$b_2 = 3a_{30}e_2 + a_{21}$

$b_1 = 2a_{21}e_2 + 2a_{12}$

$b_0 = a_{12}e_2 + 3a_{03}$

According to the catastrophe theory [1], the instability criterion of the slope is

$$w < 0 \tag{5}$$

After computing the value w from the monitoring data of a slope, the state of the rock slope may be considered to be unstable if w changes from the positive to the negative.

2.2 The determination of the initial unstable points

If the rock slope was in the unstable state determined by the elliptic umbilic catastrophe model, the critical positions, or the coordinates (x_0,y_0), can be also obtained following the method used by Liu et al [8]: $y_0=0.5v/(x_0-w)$, where is the root of Eq.(6):

$$\bar{x}^4 + Q_2\bar{x}^2 + Q_1\bar{x} + Q_0 = 0 \tag{6}$$

here

$Q_2 = -3w^2 - u$

$Q_1 = 2w^3 + 2uw$

$Q_0 = -uw^2 - u^2/2$

In this case, the initial unstable point (X_0,Y_0) may be considered as the modified values of the critical coordinates (x_0,y_0) and can be obtained by Eq.(7).

$$X_0 = K_m x_0,\ Y_0 = K_n y_0 \tag{7}$$

here

$$K_m = \xi_1x_0^2 + \xi_2y_0^2 + \xi_3\Delta z_{\max}^2 + \xi_4x_0y_0 + \xi_5x_0\Delta z_{\max} + \xi_6y_0\Delta z_{\max} + \xi_7x_0 + \xi_8y_0 + \xi_9\Delta z_{\max} + \xi_{10} \tag{8a}$$

$$K_n = \eta_1x_0^2 + \eta_2y_0^2 + \eta_3\Delta z_{\max}^2 + \eta_4x_0y_0 + \eta_5x_0\Delta z_{\max} + \eta_6y_0\Delta z_{\max} + \eta_7x_0 + \eta_8y_0 + \eta_9\Delta z_{\max} + \eta_{10} \tag{8b}$$

$$\Delta z_{\max} = x_0^3/3 - x_0y_0^2 + w(x_0^2 + y_0^2) - ux_0 + vy_0 + R_{00} \tag{8c}$$

where ξ_1, …, ξ_{10} and η_1, …, η_{10} are the curve-fit coefficients deduced from the practical monitoring data using the least square method. The detailed expressions are omitted to save the space.

3. EXAMPLES

Next we illustrate the application of the elliptic umbilic catastrophe model taking two examples: one is Baiheling landslide in China and another is Ca'di Malta landslide in Italy.

3.1 Example 1

Fig.1 shows the planar layout of Baiheling slope [10], which is located in the northwestern of Huzhou City, Zejiang Province, China. The landslides occurred in three different times: in October, 1960; in March to July, 1961; and in 1969.

The planar coordinates (x,y) and elevation differences $\triangle z$ of 10 positions between October, 1960 and July 1961 can be derived from Tang et al (2002). Then, the criterion value w is -0.738 (see Table 1). Considering Eq.(5) (here w was less than 0), the slope was in the unstable state in July, 1961. The occurrence of the landslide verified the reliability of the method presented in this paper. The initial unstable points are not predicted here due to the limitation of the monitoring data. The initial unstable points may be determined by Eqs.(6-8) if the monitoring data were available.

3.2 Example 2

Fig.2 shows the planar layout of Ca'di Malta landslide [11], which is located in the southern of Bologna and the eastern of Reno River, Italy. The bedrock of the landslide consists of Cretaceous shale. The landslide occurred in three different times: in May 30, 1914; in August, 1996; and in September, 1998. The local government used the Global Position System (GPS) to monitor the displacements of the landslide to develop a pre-cautious mechanism for the landslide (Mora et al, 2003).

The planar coordinates (x,y) and elevation differences $\triangle z$ of 10 positions can be derived from Mora et al (2003). The criterion value w is -0.738 (see Table 2). Considering Eq.(5) (here w was greater than 0), the slope was in the stable state in September, 1998. There is no landslide from September, 1998, which further shows the reliability of the proposed method.

Table 1 Computation result of Baiheling landslide

Coefficients	Values	Coefficients	Values
a_{30}	7.974E-05	e_2	0.675
a_{21}	-3.226E-06	e_1	-0.630
a_{12}	-1.320E-06	A_2	-19.867
a_{03}	-2.212E-05	B_2	41.998
a_{20}	-1.023E-02	A_1	12.516
a_{11}	3.498E-04	B_1	28.349
a_{02}	3.746E-03	c	-0.526
a_{10}	3.862E-01	d	-6.763
a_{01}	-1.882E-01	w	-0.738
a_{00}	-3.042E+00		

Table 2 Computation result of Ca'di Malta landslide

Coefficients	Values	Coefficients	Values
a_{30}	1.388E-05	e_2	-0.363
a_{21}	2.784E-06	e_1	-0.137
a_{12}	-2.069E-06	A_2	-129.328
a_{03}	-4.542E-07	B_2	56.371
a_{20}	-7.673E-03	A_1	17.718
a_{11}	-3.697E-04	B_1	-20.463
a_{02}	5.547E-04	c	-4.362
a_{10}	1.369E+00	d	-1.971
a_{01}	-4.718E-02	w	3.338
a_{00}	-7.784E+01		

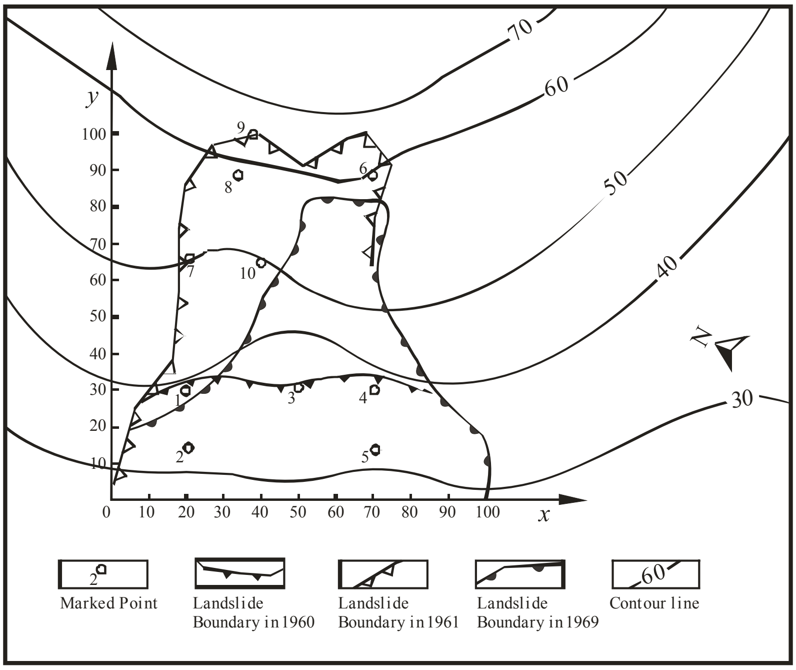

Fig. 1 Horizontal plan of Baiheling landslide

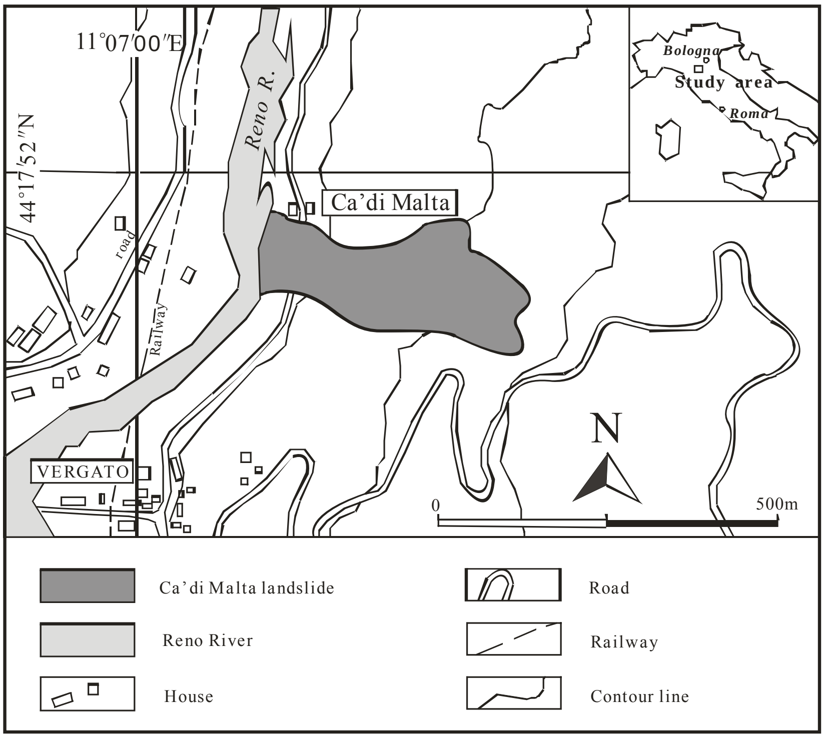

Fig.2 Horizontal plan of Ca'di Malta landslide (after Mora et al, modified)

4. CONCLUSIONS

(1)After the expression of vertical deformations with associated coordinates in rock slope was initially established and then transformed into the standard form of elliptic umbilic catastrophe model, the stability criterion and initial unstable points were determined.

(2)The computations to the Ca'di Malta landslide in Italy and the Baiheling landslide in China were performed using the developed elliptic umbilic catastrophe model. The good agreement between the predicted and practical results shows the reliability of the method presented in this paper.

(3)The proposed method may be used to forecast the stability state and initial unstable points of a slope from the monitoring data, which may provide the foundations both in dynamically predicting the stability of the slope and in efficiently treating the disaster.

ACKNOWLEDGEMENTS

Financial support for this study was provided by a grant from the Natural Sciences Foundation Committee at China under Grant No.40572162. This support is gratefully acknowledged.

REFERENCES

1) Lin, F.H. (1987): *The catastrophe theory and its applications*. Shanghai: Shanghai Jiaotong University Press (in Chinese).
2) Saunders, P.T. (Translated by Lin, F.H.) (1987): *An introduction to catastrophe theory*. Shanghai: Shanghai Literature Press of Science and Technology (in Chinese).
3) Henley, S. (1976): Catastrophe theory model in geology. *Math Geol*, **8**(6), 649-655.
4) John, M.C. and Shaw, B. (1976): The geological implication of steady-state mechanisms in catastrophe theory. *Math Geol*, **8**(6), 657-662.
5) Qin, S.Q., Jiao, J.J., Tang, C.A., et al. (2006): Instability leading to coal bumps and nonlinear evolutionary mechanisms for a coal-pillar-and-roof system. *International Journal of Solids and Structures*, **43,** 7407-7423.
6) Sun, Q., Liu, T. B., Qin, S.Q., et al. (2006): The swallow-tail catastrophe theory for the slope instability. *Chinese Journal of Engineering Geology*, **14**(6), 852-855(in Chinese).
7) Xu, J.C., Shang, Y.Q., Zheng, S.N., et al. (2005): The cusp catastrophe theory of the shallow landslide in intensely infiltration. *Zhejiang University Journal* (Technology Division), **39**(11), 1675-1679(in Chinese).
8) Long, H., Qin, S.Q., Wan, Z.Q., et al. (2002): The cusp catastrophe theory of landslide triggered by rainfall. *Chinese Journal Roch Mechanics and Engineering*, **21**(4), 502-508 (in Chinese).
9) Liu, D.W., Wang, Y.J., Lu, J.L., et al. (1994): The elliptic umbilic catastrophe model for determination of dangerous place and magnitude of intensive earthquakes. *Chinese Journal of Geophysics*, **37**(Suppl. II), 177-184(in Chinese).
10) Tang, Y.Q., Shi, Z.M., Dong, B.Y., et al. (2002): *Practice course for engineering geology*. Shanghai: Tongji University Press (in Chinese).
11) Mora, P., Baldi P., Casula G., et al. (2003): Global Positioning Systems and digital photogrammetry for the monitoring of mass movements: application to the Ca'di Malta landslide(northern Apennines, Italy). *Engineering Geology*, **68,** 103-121.